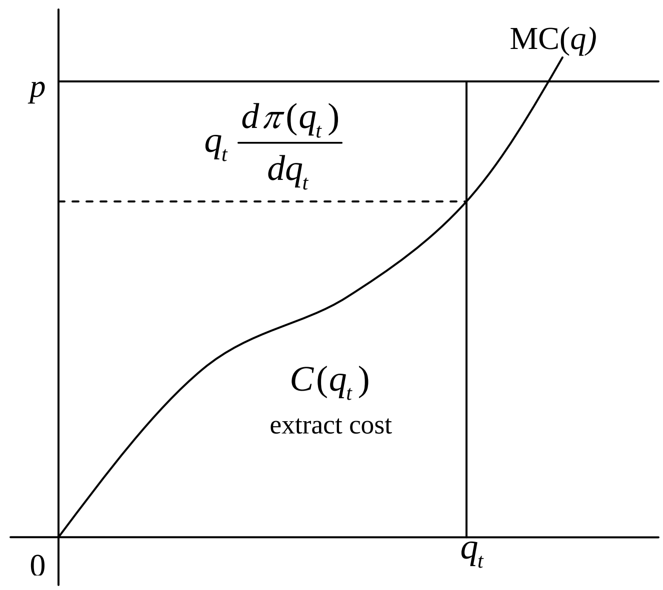
Sustainable Investment Policy

- An oil exporting nation has interest rF_t plus profit $\pi(q_t)$ to spend each period on consumption C_t plus investment I_t.
- When I_t is equal to current exhaustible resource rent $q_t \dfrac{d\pi(q_t)}{dq_t}$ and I_t goes into fund $F_t \left(\dfrac{dF_t}{dt} = I_t \right)$, then consumption is unchanging. $\left(\text{Note: } \dfrac{d\pi(q)}{dq} = p - MC(q) \right)$



$$C_t = rF_t + [q_t p - C(q_t)] - I_t$$

$$\frac{dC_t}{dt} = r\frac{dF_t}{dt} + [p - MC(q_t)]\frac{dq_t}{dt} - \frac{dI_t}{dt} \qquad\qquad \frac{dI_t}{dt} = \dot{q}_t \frac{d\pi(q_t)}{dq_t} + \frac{d\left[\frac{d\pi(q_t)}{dq_t}\right]}{dt} q_t$$

$$= rI_t - q_t \frac{d\left[\frac{d\pi(q_t)}{dq_t}\right]}{dt}$$

$$= rq_t \frac{d\pi(q_t)}{dq_t} - q_t \frac{d\left[\frac{d\pi(q_t)}{dq_t}\right]}{dt}$$

$$= [0] q_t \qquad\qquad r\% \text{ rule is } \frac{d\left[\frac{d\pi(q_t)}{dq_t}\right]}{dt} = \frac{d\pi(q_t)}{dq_t} r$$

<u>Extractive Firm with S_t Tons Remaining</u>

Profit from sale of q_t in period t is $pq_t - C(q_t) = \pi(q_t)$
Present value of profit is,

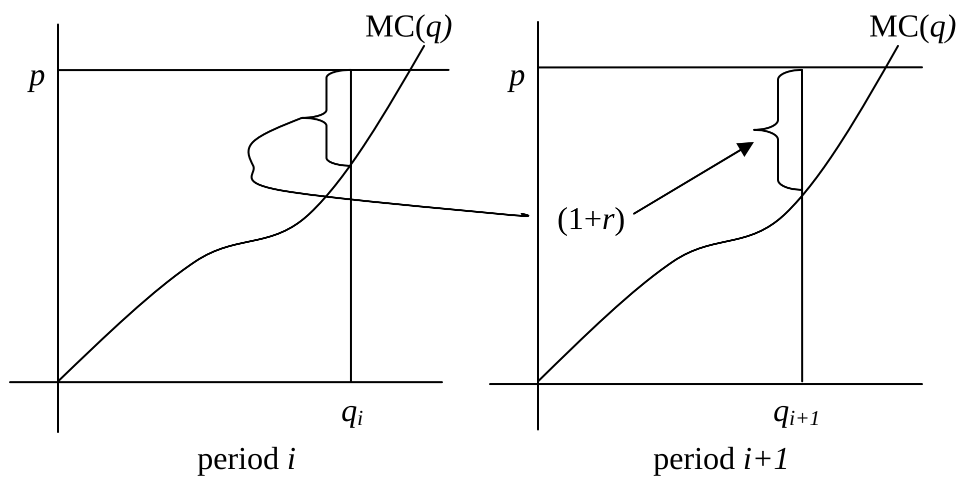
$$V_t = \pi(q_t) + \left(\frac{1}{1+r}\right)\pi(q_{t+1}) + \left(\frac{1}{1+r}\right)^2 \pi(q_{t+2}) + \ldots + \left(\frac{1}{1+r}\right)^T \pi(q_T)$$

$$q_t + q_{t+1} + q_{t+2} + \ldots + q_T = S_t, \quad \text{remaining stock}$$

maximize V_t by choice of path of q_i's implies

$$\frac{\dfrac{d\pi(q_i)}{dq_i} - \dfrac{d\pi(q_{i-1})}{dq_{i-1}}}{\dfrac{d\pi(q_{i-1})}{dq_{i-1}}} = r$$

percentage change in marginal profit equals r%



Last Period, *T*

$$\frac{d[pq_T - C(q_T)]}{dq_T} = \frac{pq_T - C(q_T)}{dq_T}$$

marginal profit from q_T = Average profit from q_T